Penalized Model Selection and Asymptotic Minimaxity

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Abstract

The last decade witnesses a summit of research on the penalized model selection, which, as a method of adaptive estimation and prediction, has immensely broad applications. Fruitful progresses have been achieved in seeking good penalty procedures from different aspects. It is important to study the procedures from the minimax optimality point of view because minimax theory can provide a deep understanding and an organizing scheme for many statistical developments. We investigate properties of a class of model selection procedures in the multivariate normal mean estimation problem, which is closely related to hypothesis testing and is connected with more sophisticated models through the asymptotic equivalence. Specifically, we obtain sharp minimax results for the estimation of an n-dimensional normal mean under quadratic loss. The estimators are chosen by penalized least squares with a penalty that grows like $ck\log(n/k)$, for k equal to the number of nonzero elements in the estimating vector. For a wide range of sparse parameter spaces, we show that this penalized estimator adaptively achieves the sharp minimax rate (with the exactly correct multiplication constant) if and only if $c = 2$. Thus we establish that a conjecture by Abramovich, Benjamini, Donoho and Johnstone (2006) is true. When $c > 2$, this penalized estimator is minimax optimal up to a constant whose formula is also given. Our results unify the theory obtained by many other authors for penalized estimation of normal means.